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Path-integral quantization of electrodynamics in dielectric media

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Abstract. In the present paper we study the Faddeev–Popov path-integral quantization of electrodynamics in an inhomogeneous dielectric medium. We quantize all polarizations of the photons and introduce the corresponding ghost fields. Using the heat kernel technique, we express the heat kernel coefficients in terms of the dielectricity $\epsilon(x)$ and calculate the ultraviolet divergent terms in the effective action. No cancellation between ghosts and 'non-physical' degrees of freedom of the photon is observed.

1. Introduction

The Casimir effect describes the forces resulting from the vacuum fluctuations (ground state energy) of the electromagnetic field in simple situations realized by conducting surfaces. These forces can be viewed as retarded Van der Waals forces between the atoms constituting the surfaces (and those within). As a generalization of this picture one can consider some medium, which can be characterized either by atoms at positions x_i with their individual polarizabilities α_i or by a macroscopic permittivity $\epsilon(x)$ and permeability $\mu(x)$. Again, we can calculate the resulting potential of the Van der Waals forces or the vacuum energy $E_0[\epsilon(x), \mu(x)]$ of the electromagnetic field in a background given by $\epsilon(x)$ (respectively $\mu(x)$). Taking into account that real permittivity (respectively permeability) are functions of the photon frequency we arrive at the problem of calculating $E_0[\epsilon(x, \omega), \mu(x, \omega)]$. The dependence on ω has as a physical background, besides others, the observation that any medium becomes transparent for ω sufficiently high (we do not consider inelastic effects here). Therefore $\epsilon, \mu \to 1$ for $\omega \to \infty$ should serve as a natural ultraviolet regularization. This is widely believed, but not shown in a rigorous way as yet.

The problem of the calculation of $E_0[\epsilon(x), \mu(x)]$, i.e. without frequency dependence, may well be posed independently. A physical justification could be that the essential contribution results after a proper renormalization from quite low frequencies ω, where ϵ and μ can be viewed as approximately independent of ω. In that case we do not have a natural regularization and have to proceed as in the general situation with sharp boundary conditions or a general background field. For technical reasons we use the zeta-functional regularization. Then the first step is to calculate the divergent contributions (the proper

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¶ e-mail address: vassil@snoopy.niif.spb.su technical tool being the heat kernel expansion); the second step is to formulate a model for the interpretation of the renormalization (this is to be able to reinterpret the subtraction of the divergences as a renormalization of classical quantities such as volume, surface tension, etc, as discussed in [1] or like the mass and the coupling constant of the background field as discussed in [2]) and, finally, in a third step to calculate the renormalized ground state energy E_0. In the present paper we carry out the first step and discuss the second to some extent.

To a large extent our technique is borrowed from the background field formalism in gauge theories which was developed long ago (see [3–5]). There are, however, two important differences. First, in our case the background field $\epsilon(x)$ is not quantized. Therefore, the ordinary renormalization procedure is not applicable. Second, Lorentz invariance is broken, and, hence, we must consider the Hamiltonian quantization approach to define a proper path-integral measure.

The forces resulting from the electromagnetic vacuum fluctuations in polarizable media have been given much attention. The common features of most of these investigations are sharp boundaries separating regions of different values of $\epsilon(x)$ and simple geometries (planes, cylinders and so on) as well as frequency-dependent $\epsilon(\omega)$. A basic paper is [6] by Candelas. For a dielectric sphere and cylinder, for example, calculations were performed in [7, 8] (and earlier work cited therein). For a medium with inhomogeneous dielectric and diamagnetic constants a perturbative analysis of the divergent contributions to the Casimir energy up to the second order is given in [9] (actually contributions up to fourth order appear to be divergent, cf formula (21) below). An interesting review on the topic can be found in the book by Bermann [10]. Also, much attention had been spent on a possible explanation of sonoluminescence as a dynamical Casimir effect, especially in a series of papers by Schwinger [11]. Recently, the bulk and surface energy contributions have been discussed [12, 13].

However, with respect to the renormalization it is difficult to deal with sharp boundaries (respectively non-smooth background fields). It is known that additional contributions to the heat kernel expansion occur and therefore additional counterterms result for which a general theory is still missing. Therefore we restrict ourselves in the present paper to $\epsilon(x)$ which are smooth functions on x.

There is still another problem we have to pay attention to. In the common understanding the quantization of QED in media is performed in the Coulomb gauge, i.e. the two 'physical' polarizations of the photon are quantized. Also, there are known procedures where all polarizations of the photon are quantized and the gauge invariance (in the presence of boundaries) is restored by ghosts which also have to fulfil boundary conditions (one of the first is [14], later on it was discussed in [15]). In most cases their contributions cancel that resulting from the 'unphysical' photons, but counterexamples are known (e.g. for QED in curved spacetime, [16]). Within the framework of quantum optics the canonical quantization of photons was considered in [18] without, however, analysing the ghost contributions. An alternative approach for quantization in covariant gauge without ghosts, however restricted to sharp boundaries, had been developed in [17].

In the present paper we analyse the problem of QED with a position-dependent permittivity $\epsilon(x)$ from the point of view of general quantum gauge theory in an external field. We analyse the canonical path-integral measure and the corresponding configuration space measure. A gauge-fixing term is introduced together with the ghost action. Next we analyse the ultraviolet structure of the theory by means of the heat kernel expansion. No cancellation between ghosts and photon modes is obtained.

Our paper is organized as follows. In the next section the quantization of the theory is considered and the path integral is derived. In section 3 we use the heat kernel expansion to evaluate ultraviolet divergences. Concluding remarks are given in section 4. An appendix contains an alternative calculation to check the results of section 3.

2. Canonical quantization and gauge choice

Consider the action for the electromagnetic field in a dielectric media with permittivity $\epsilon(x)$:

$$S = \int \mathrm{d}^4x \, \tfrac{1}{2}\big(\epsilon(x)E^2 - B^2\big). \tag{1}$$

To avoid technical complexities we put the permeability $\mu = 1$ and assume that ϵ depends on spatial coordinates only.

Let us rewrite the action (1) in the canonical first-order form:

$$S_1 = \int \mathrm{d}^4x \left(P^i\, \partial_0 A_i + A_0 \partial_i P^i - \frac{1}{\epsilon(x)} P^i P^i - \frac{1}{2} B^2 \right). \tag{2}$$

Here A_μ is the vector potential and $P^i = -\epsilon(x)\, E_i$ is the momentum conjugate to A_i. Canonical Poisson brackets are

$$\big\{A_i(\boldsymbol{x}, t), P^j(\boldsymbol{y}, t)\big\} = \delta_i^j \delta(\boldsymbol{x} - \boldsymbol{y}). \tag{3}$$

The same brackets were obtained in [18]. A_0 plays the role of a Lagrange multiplier generating the Gauss law constraint, which in turn generates gauge transformations. According to the general method [19] of quantization of gauge theories we can write down the path integral

$$Z = \int \mathrm{D}A_i \, \mathrm{D}A_0 \, \mathrm{D}P^j \, J_{\mathrm{FP}} \delta(\chi(A_i)) \exp(\mathrm{i}S_1) \tag{4}$$

where $\chi(A_i)$ is a gauge-fixing condition, J_{FP} is the Faddeev–Popov determinant, $J_{\mathrm{FP}} = \det\{\chi(A), \partial_j P^j\}$. Now we can perform the integration over the momenta P^j. It produces the factor $\prod_x \sqrt{\epsilon(x)^3}$ which should be absorbed in the path-integral measure $\mathrm{D}A_i$. We arrive at the following expression:

$$Z = \int \mathrm{D}\tilde{A}_i \, \mathrm{D}A_0 \, J_{\mathrm{FP}} \delta(\chi(A)) \exp(\mathrm{i}S) \qquad \tilde{A}_i = \sqrt{\epsilon} A_i. \tag{5}$$

Our $\tilde{A}$ variables coincide with the q' variables of Glauber and Lewenstein [18]. Note that the measure in (5) differs from the naive one $\prod \mathrm{D}A_\mu$. We can use the Faddeev–Popov trick to transform the path integral (5) to whatever gauge condition we prefer, introduce a gauge-fixing term and ghost fields. There is nothing specific in this respect in the present model. All steps repeat those of a standard textbook [19]. The result is

$$\begin{aligned} Z = \int \mathrm{D}\tilde{A}_i \, \mathrm{D}A_0 \, \mathrm{D}c \, \mathrm{D}\bar{c} \exp \bigg\{ \mathrm{i} \int \mathrm{d}^4x \big[\tfrac{1}{4} \big(2\epsilon(x) \big(\partial_0 \epsilon^{-1/2} \tilde{A}_i - \partial_i A_0 \big)^2 \\ - \big(\partial_i \epsilon^{-1/2} \tilde{A}_k - \partial_k \epsilon^{-1/2} \tilde{A}_i \big)^2 \big) + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{ghost}} \big] \bigg\} \end{aligned} \tag{6}$$

where $\mathcal{L}_{\mathrm{gf}}$ and $\mathcal{L}_{\mathrm{ghost}}$ are the gauge-fixing term and ghost action, respectively. As usual, we can bring the action in (6) into the form $\int A_\mu L_{\mu\nu} A_\nu$, where $L_{\mu\nu}$ is a second-order differential operator. In calculating the effective action and the heat kernel expansion it is much more convenient to deal with operators of Laplace type, i.e. operators with scalar leading symbol. There is a unique gauge choice which splits the $L_{\mu\nu}$ into a direct sum of operators of Laplace type. This choice is

$$\begin{aligned}\mathcal{L}_{\rm gf} &= -\tfrac{1}{2}\left(\epsilon^{-1}\partial_i\epsilon^{1/2}\tilde{A}_i - \epsilon\partial_0 A_0\right)^2 \\ \mathcal{L}_{\rm ghost} &= -\bar{c}\left(-\epsilon^{-1}\partial_i\epsilon\partial_i + \epsilon\partial_0^2\right)c.\end{aligned} \tag{7}$$

The action for the electromagnetic field A then takes the form

$$\begin{aligned}\tfrac{1}{2}\int \mathrm{d}^4x\, &\left[\epsilon(\partial_i A_0)^2 - \epsilon^2(\partial_0 A_0)^2 + (\partial_0\tilde{A}_i)^2\right.\\ &\left.+\tilde{A}_i\epsilon^{-1/2}\left(\partial_j^2\delta_{ik} - e_i\partial_k + \partial_i e_k - e_i e_k\right)\epsilon^{-1/2}\tilde{A}_k\right] \qquad e_i = \partial_i \ln\epsilon.\end{aligned} \tag{8}$$

Note, that the mixing between A_0 and $\tilde{A}_i$ is removed completely.

The total action with gauge fixing and the ghost term is invariant under the BRST transformations with the parameter $\sigma(x)$:

$$\begin{aligned}\delta A_0 &= \partial_0\sigma c\\ \delta\tilde{A}_i &= \epsilon^{1/2}\partial_i\sigma c\\ \delta c &= 0\\ \delta\bar{c} &= \left(-\epsilon^{-1}\partial_i\epsilon^{1/2}\tilde{A}_i + \epsilon\partial_0 A_0\right)\sigma\end{aligned} \tag{9}$$

which are given here to complete the picture.

3. Effective action and heat kernel expansion

Now we are able to integrate over A_0, $\tilde{A}$ and the ghosts. The resulting path integral reads, after Wick rotation to the Euclidean domain,

$$Z = Z[A_0]\,Z[\tilde{A}]\,Z[\bar{c}, c] \tag{10}$$

where the separate contributions are of the form

$$\begin{aligned}Z[A_0] &= \det{}^{-1/2}\left(-\partial_i\epsilon\partial_i - \epsilon^2\partial_0^2\right)\\ Z[\tilde{A}] &= \det{}^{-1/2}\left(-\frac{1}{\epsilon}\partial_k^2\delta_{ij} - \partial_0^2\delta_{ij} - G_i\partial_j + G_j\partial_i - M_{ij}\right)\\ Z[\bar{c}, c] &= \det\left(-\epsilon^{-1}\partial_i\epsilon\partial_i - \epsilon\partial_0^2\right).\end{aligned} \tag{11}$$

We introduced the notation

$$G_i = \frac{e_i}{\epsilon} \qquad M_{ij} = \frac{1}{\epsilon}(e_{ij} - e_i e_j) \qquad e_{ij} = \partial_i e_j. \tag{12}$$

For the functional determinants we use the integral representation

$$\log\det(L) = \int_0^\infty \frac{\mathrm{d}t}{t} K(L;t) \tag{13}$$

where the heat kernel $K(L;t)$ for a second-order elliptic operator L is

$$K(L;t) = \mathrm{Tr}\exp(-tL). \tag{14}$$

The ultraviolet behaviour of functional determinants is given by the asymptotic expansion of the heat kernel (14) as $t \to +0$. Since all the operators are of Laplace type, we can use the general theory [20]. Each of the operators has the structure

$$L = -\left(g^{\mu\nu}\partial_\mu\partial_\nu + a^\sigma\partial_\sigma + b\right) \tag{15}$$

where $g^{\mu\nu}$ plays the role of a metric. a^σ and b are local sections of the endomorphism $\mathrm{End}(V)$ of a certain vector bundle. By introducing a connection ω_μ in the vector bundle V, one can bring L into the form

$$L = -\left(g^{\mu\nu}\nabla_\mu\nabla_\nu + E\right) \tag{16}$$

where ∇ is a sum of the Riemannian covariant derivative with respect to the metric g and the connection ω. The explicit form of ω and E is

$$\begin{aligned} \omega_\delta &= \tfrac{1}{2} g_{\nu\delta}\left(a^\nu + g^{\mu\sigma}\Gamma^\nu_{\mu\sigma}\right) \\ E &= b - g^{\mu\nu}\left(\partial_\mu\omega_\nu + \omega_\mu\omega_\nu - \omega_\sigma\Gamma^\sigma_{\mu\nu}\right). \end{aligned} \tag{17}$$

As usual, Γ denotes the Christoffel connection.

Given the geometric quantities g, ω and E, we are able to calculate the coefficients a_n of the asymptotic expansion

$$\mathrm{Tr}(f\exp(-tL)) = t^{-2}\sum_{n=0}^{\infty} t^n a_n(f, L) \tag{18}$$

for a function f. The coefficients $a_n(f, L)$ contain information on the asymptotics of the heat kernel diagonal $\langle x|\exp(-tL)|x\rangle$. The analytical expressions for the first coefficients are known [20]:

$$\begin{aligned} a_0 &= \frac{1}{(4\pi)^2}\,\mathrm{tr}_V\int \mathrm{d}^4x\, g^{1/2} f \\ a_1 &= \frac{1}{(4\pi)^2}\,\mathrm{tr}_V\int \mathrm{d}^4x\, g^{1/2} f\left(E + \tfrac{1}{6}\tau\right) \\ a_2 &= \frac{1}{(4\pi)^2}\,\mathrm{tr}_V\int \mathrm{d}^4x\, g^{1/2}\tfrac{1}{360} f\big(60E_{;\mu}{}^{\mu} + 60\tau E + 180E^2 + 30\Omega_{\mu\nu}\Omega^{\mu\nu} \\ &\qquad + 12\tau_{;\mu}{}^{\mu} + 5\tau^2 - 2\rho^2 + 2R^2\big). \end{aligned} \tag{19}$$

Here R, ρ and τ are the Riemann tensor, Ricci tensor and scalar curvature of the metric g, respectively. A semicolon denotes covariant differentiation, $E_{;\mu} = \nabla_\mu E$. All indices are lowered and raised with the metric tensor, tr_V is the bundle (matrix) trace, Ω is the field strength of the connection ω:

$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu + \omega_\mu\omega_\nu - \omega_\nu\omega_\mu. \tag{20}$$

The three coefficients (19) are enough to describe the one-loop ultraviolet divergences in four-dimensional quantum field theory in an infinite spacetime.

Now our problem is reduced to the calculation of the geometric quantities appearing in (19). For the ghost operator we have

$$\begin{aligned}
&g_{ij} = \delta_{ij} \qquad g_{00} = \epsilon^{-1}(x)\\
&\Gamma^i_{00} = \frac{1}{2\epsilon} e_i \qquad \Gamma^0_{0i} = -\tfrac{1}{2} e_i\\
&\omega_0 = 0 \qquad \omega_i = \tfrac{3}{4} e_i\\
&E = -\tfrac{3}{4} e_{ii} - \tfrac{3}{16} e_i e_i\\
&R^i{}_{jkl} = 0\\
&R^0{}_{i0j} = -\tfrac{1}{2} e_{ij} + \tfrac{1}{4} e_i e_j\\
&\rho_{ij} = \tfrac{1}{2} e_{ij} - \tfrac{1}{4} e_i e_j\\
&\rho_{00} = \frac{1}{2\epsilon}\left(e_{ii} - \tfrac{1}{2} e_j e_j\right)\\
&\tau = e_{ii} - \tfrac{1}{2} e_i e_i .
\end{aligned} \tag{21}$$

For the operator acting on A_0 the relevant quantities are

$$\begin{aligned}
&g_{ij} = \epsilon^{-1}\delta_{ij} \qquad g_{00} = \epsilon^{-2}(x)\\
&\Gamma^i_{00} = \frac{1}{\epsilon} e_i \qquad \Gamma^0_{0i} = -e_i\\
&\Gamma^k_{ij} = -\tfrac{1}{2}(e_i\delta_{jk} + e_j\delta_{ik} - e_k\delta_{ij})\\
&\omega_0 = 0 \qquad \omega_i = \tfrac{5}{4} e_i\\
&E = \epsilon\left(-\tfrac{5}{4} e_{ii} + \tfrac{5}{16} e_i e_i\right)\\
&R^i{}_{jkl} = \tfrac{1}{2}(-e_{jl}\delta_{ik} + e_{jk}\delta_{il} + e_{il}\delta_{kj} - e_{ik}\delta_{lj})\\
&\qquad\qquad + \tfrac{1}{4}(e_p e_p(\delta_{jl}\delta_{ki} - \delta_{jk}\delta_{li}) + e_k e_j \delta_{li} - e_k e_i \delta_{jl} - e_l e_j \delta_{ki} + e_l e_i \delta_{jk})\\
&R^0{}_{i0j} = -e_{ij} + \tfrac{1}{2} e_l e_l \delta_{ij}\\
&\rho_{ij} = \tfrac{3}{2} e_{ij} + \tfrac{1}{2}\delta_{ij} e_{kk} + \tfrac{1}{4} e_i e_j - \tfrac{3}{4}\delta_{ij} e_k e_k\\
&\rho_{00} = \frac{1}{\epsilon}\left(e_{ii} - \tfrac{3}{2} e_j e_j\right)\\
&\tau = \epsilon\left(4 e_{ii} - \tfrac{7}{2} e_i e_i\right).
\end{aligned} \tag{22}$$

For the operator acting on $\tilde{A}$ we obtain

$$\begin{aligned}
&g_{ij} = \epsilon\delta_{ij} \qquad g_{00} = 1\\
&\Gamma^k_{ij} = \tfrac{1}{2}(e_i\delta_{jk} + e_j\delta_{ik} - e_k\delta_{ij})\\
&\omega^{ab}_l = \tfrac{1}{2}\left(-e_a\delta_{bl} + e_b\delta_{al} - \tfrac{1}{2} e_l \delta_{ab}\right)\\
&E_{ab} = M_{ab} + \frac{1}{4\epsilon}\left(e_{kk}\delta_{ab} + e_a e_b + \tfrac{5}{4} e_p e_p \delta_{ab}\right)\\
&R^i{}_{jkl} = \tfrac{1}{2}(e_{jl}\delta_{ik} - e_{jk}\delta_{il} - e_{il}\delta_{kj} + e_{ik}\delta_{lj})\\
&\qquad\qquad + \tfrac{1}{4}(e_p e_p(\delta_{jl}\delta_{ki} - \delta_{jk}\delta_{li}) + e_k e_j \delta_{li} - e_k e_i \delta_{jl} - e_l e_j \delta_{ki} + e_l e_i \delta_{jk})\\
&\rho_{jk} = -\tfrac{1}{2}(e_{jk} + e_{pp}\delta_{jk}) + \tfrac{1}{4}(e_k e_j - e_p e_p \delta_{kj})\\
&\tau = \frac{1}{\epsilon}\left(-2 e_{pp} - \tfrac{1}{2} e_p e_p\right).
\end{aligned} \tag{23}$$

Here for convenience we prefer to keep the distinction between coordinate indices $\{i, j, k, l\}$ and bundle indices $\{a, b\}$, though they all run from 1 to 3. In equations (21)–(23) repeated indices are contracted with the flat space metric δ_{ij}.

It is instructive to express the heat kernel coefficients in terms of ϵ and its derivatives:

$$
\begin{aligned}
K_{\mathrm{gh}}(f,t) = \frac{1}{(4\pi t)^2}\int \mathrm{d}^4x\,\epsilon^{-1/2} f\big\{1 + t\big(-\tfrac{7}{12}e_{ii} - \tfrac{13}{48}e_ie_i\big)\\
+\tfrac{1}{360}t^2\big(-33e_{iijj} - 18e_ie_{ijj} - 33e_{ij}e_{ij} + \tfrac{237}{4}e_{ii}e_{jj}\\
+\tfrac{531}{8}e_{ij}e_ie_j + \tfrac{33}{4}e_{ii}e_je_j + \tfrac{837}{64}e_ie_ie_je_j\big) + \mathrm{O}\big(t^3\big)\big\}\\
K_{[A_0]}(f,t) = \frac{1}{(4\pi t)^2}\int \mathrm{d}^4x\,\epsilon^{-5/2} f\big\{1 + t\epsilon\big(-\tfrac{7}{12}e_{ii} - \tfrac{13}{48}e_ie_i\big)\\
+\tfrac{1}{360}t^2\epsilon^2\big(-27e_{iijj} - 60e_ie_{ijj} - 41e_{ij}e_{ij} + \tfrac{119}{4}e_{ii}e_{jj}\\
-\tfrac{91}{8}e_{ij}e_ie_j + \tfrac{415}{8}e_{ii}e_je_j + \tfrac{4141}{64}e_ie_ie_je_j\big) + \mathrm{O}\big(t^3\big)\big\}\\
K_{[\tilde{A}]}(f,t) = \frac{1}{(4\pi t)^2}\int \mathrm{d}^4x\,\epsilon^{3/2} f\bigg\{1 + \frac{t}{\epsilon}\big(\tfrac{3}{4}e_{ii} - \tfrac{1}{16}e_ie_i\big)\\
+\frac{t^2}{360\epsilon^2}\big(81e_{iijj} - 111e_ie_{ijj} + 162e_{ij}e_{ij} - \tfrac{711}{4}e_{ii}e_{jj}\\
-\tfrac{1029}{4}e_{ij}e_ie_j + \tfrac{793}{8}e_{ii}e_je_j + \tfrac{4263}{16}e_ie_ie_je_j\big) + \mathrm{O}\big(t^3\big)\bigg\}.
\end{aligned}
\tag{24}
$$

Here $e_{i\ldots j} = \partial_i \ldots \partial_j \ln\epsilon$. This completes the calculation of the UV divergent terms.

We can define a 'total' heat kernel as $K_{[A_0]} + K_{[\tilde{A}]} - 2K_{\mathrm{gh}}$. We see that the contribution of ghosts is not cancelled by that of A_0 and of the 'non-physical' components of $\tilde{A}$.

As a check, in the appendix we derive (24) by an alternative method.

The asymptotic expansion constructed above gives $2n$ spatial derivatives of ϵ in any a_n. Hence it is clear that a certain smoothness of $\epsilon(x)$ is needed. Our expansion is not valid if ϵ changes abruptly, such as, for example, for a bubble in water. For the configurations of the latter type, boundary terms in the heat kernel expansion should be taken into account.

4. Conclusions and discussion

In this paper we have performed the path-integral quantization of electromagnetic fields in a dielectric medium. As a first step, we considered the first-order action and derived the canonical Poisson brackets. Next, we constructed the canonical (simplectic) measure in the phase space. We built up a measure in the configuration space by means of an integration over the canonical momenta. This measure appeared to be different from the naive one. By choosing a suitable gauge-fixing condition (7) we reduced the path integral to a product of three determinants of operators of Laplace type. For the evaluation of the ultraviolet divergent parts of these determinants the standard heat kernel technique [20] is available. Our results are re-checked by another technique (see the appendix). We observed no cancellation of ultraviolet divergencies between ghosts and any 'non-physical' components of the vector potential. Thus it is highly unlikely that the full quantized electrodynamics in dielectric media is equivalent to a theory where only two polarizations of photons are quantized.

In principle, the divergences of the effective action can depend on the choice of gauge. However, if a gauge-invariant regularization such as the zeta function one is used, the pole term proportional to the t^0 term in the total heat kernel must be gauge independent. Therefore, our main result is not sensitive to the particular method of gauge fixing.

The next step is to work out a suitable cut-off procedure for the path integral. This problem is very non-trivial in the present case. Since $\epsilon \to 1$ at high frequencies, the cut-off is *physical*, it will not be removed after a renormalization. Therefore, we must be sure that the basic properties of the quantum field theory, such as unitarity and the absence of gauge anomaly, are valid at a finite cut-off. After having solved this problem, it will be possible to consider the vacuuum energy densities and other physical quantities of interest.

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Appendix

In this appendix we briefly describe an alternative method for the evaluation of the heat kernel expansion which we used to check our results.

We can represent the functional trace in the right-hand side of (14) as an integral over x of diagonal matrix elements between $\langle x|$ and $|x\rangle$ and insert 'unity' expressed via an integral of momentum eigenstates:

$$\mathrm{Tr}\exp(-tL) = \int \frac{\mathrm{d}^4x\,\mathrm{d}^4k}{(2\pi)^4}\langle x|\exp(-tL)|k\rangle\langle k|x\rangle. \tag{A1}$$

The generic form of the matrix element in (A1) is $\langle x|F_1(\epsilon,\partial\epsilon)\,F_2(\partial)|k\rangle$, where F_1 and F_2 are some polynomials of ϵ and its derivatives and of ∂_i, respectively. Acting on the left F_1 is replaced by its value at the point x. Acting on the right, F_2 is replaced by $F_2(\mathrm{i}k)$. It is easy to see that the result is

$$\int \frac{\mathrm{d}^4x\,\mathrm{d}^4k}{(2\pi)^4}\exp(-tL(\epsilon(x),\partial_\mu\to\partial_\mu+\mathrm{i}k_\mu)) \tag{A2}$$

where we should take all external fields at the point x, shift all derivatives by ik, and drive derivatives to the right. It is understood, that ∂ at the very right-hand position vanishes.

Consider the heat kernel for the ghost operator:

$$K_{\mathrm{gh}}(t) = \int \frac{\mathrm{d}^4x\,\mathrm{d}^4k}{(2\pi)^4}\exp\left(t\left(\partial_i^2 + 2ik_j\partial_j + (\partial_j\log\epsilon)\partial_j + \mathrm{i}k_j(\partial_j\log\epsilon) - k^2 - \epsilon\omega^2\right)\right) \tag{A3}$$

where $\{k_\mu\} = \{\omega, k_j\}$. Time derivatives are dropped because $\epsilon(x)$ is static.

To obtain a small-t asymptotic expansion of (A3), one should isolate the factor $\exp(-t(k^2+\epsilon\omega^2))$ and expand the rest of the expression in a power series of the operators and functions involved. Next one should integrate over momenta and collect all terms with the same powers of proper time t. Denote the exponential in (A3) as $\exp(A+B)$, where $A = -t(k^2+\epsilon\omega^2)$. Note, that A does not commute with B. However, the repeated commutator $[[[B, A], A], A]$ vanishes. This allows us to present the exponential as follows (see e.g. [21]):

$$\exp(A+B) = \exp A\bigl(1 + B + \tfrac{1}{2}[B, A] + \tfrac{1}{6}[[B, A], A] + \tfrac{1}{2}B^2 + \tfrac{1}{2}[B, A]B + \tfrac{1}{6}[B, [B, A]] + \tfrac{1}{8}[B, A]^2 + \cdots\bigr). \tag{A4}$$

We retained all the terms which contribute to the two leading terms of the asymptotic expansion proportional to t^{-2} and t^{-1}.

Acting as explained above we obtain the asymptotic expansions for the heat kernels K_{gh}, $K_{[A_0]}$ and $K_{[\tilde{A}]}$. The first two terms are in complete agreement with (24). Calculations of the third terms are too complicated to be done just as a check.

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